

ADS/CFT-INSPIRED UNIFICATION AT ABOUT 4 TEV

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The use of the AdS/CFT correspondence to arrive at quiver gauge field theories is discussed, focusing on the orbifolded case without supersymmetry. An abelian orbifold with the finite group Z_p can give rise to a $G = SU(N)^p$ gauge group with chiral fermions and complex scalars in different bi-fundamental representations of G . The precision measurements at the Z resonance suggest the values $p = 12$ and $N = 3$, and a unifications scale $M_U \sim 4$ TeV.

1. Quiver Gauge Theory

The relationship of the Type IIB superstring to conformal gauge theory in $d = 4$ gives rise to an interesting class of gauge theories. Choosing the simplest compactification¹ on $AdS_5 \times S_5$ gives rise to an $N = 4$ $SU(N)$ gauge theory which is known to be conformal due to the extended global supersymmetry and non-renormalization theorems. All of the RGE β -functions for this $N = 4$ case are vanishing in perturbation theory. It is possible to break the $N = 4$ to $N = 2, 1, 0$ by replacing S_5 by an orbifold S_5/Γ where Γ is a discrete group with $\Gamma \subset SU(2), \subset SU(3), \not\subset SU(3)$ respectively.

In building a conformal gauge theory model^{2,3,4}, the steps are: (1) Choose the discrete group Γ ; (2) Embed $\Gamma \subset SU(4)$; (3) Choose the N of $SU(N)$; and (4) Embed the Standard Model $SU(3) \times SU(2) \times U(1)$ in the resultant gauge group $\bigotimes SU(N)^p$ (quiver node identification). Here we shall look only at abelian $\Gamma = Z_p$ and define $\alpha = \exp(2\pi i/p)$. It is expected from the string-field duality that the resultant field theory is conformal in the $N \rightarrow \infty$ limit, and will have a fixed manifold, or at least a fixed point, for N finite.

Before focusing on $N = 0$ non-supersymmetric cases, let us first examine an $N = 1$ model first put forward in the work of Kachru and Silverstein⁵. The choice is $\Gamma = Z_3$ and the **4** of $SU(4)$ is $\mathbf{4} = (1, \alpha, \alpha^2)$. Choosing

$N=3$ this leads to the three chiral families under $SU(3)^3$ trinification⁶

$$(3, \bar{3}, 1) + (1, 3, \bar{3}) + (\bar{3}, 1, 3) \quad (1)$$

2. Gauge Couplings.

An alternative to conformality, grand unification with supersymmetry, leads to an impressively accurate gauge coupling unification⁷. In particular it predicts an electroweak mixing angle at the Z-pole, $\sin^2\theta = 0.231$. This result may, however, be fortuitous, but rather than abandon gauge coupling unification, we can rederive $\sin^2\theta = 0.231$ in a different way by embedding the electroweak $SU(2) \times U(1)$ in $SU(N) \times SU(N) \times SU(N)$ to find $\sin^2\theta = 3/13 \simeq 0.231$ ^{4,8}. This will be a common feature of the models in this paper.

3. 4 TeV Grand Unification

Conformal invariance in two dimensions has had great success in comparison to several condensed matter systems. It is an interesting question whether conformal symmetry can have comparable success in a four-dimensional description of high-energy physics.

Even before the standard model (SM) $SU(2) \times U(1)$ electroweak theory was firmly established by experimental data, proposals were made^{9,10} of models which would subsume it into a grand unified theory (GUT) including also the dynamics¹¹ of QCD. Although the prediction of $SU(5)$ in its minimal form for the proton lifetime has long ago been excluded, *ad hoc* variants thereof¹² remain viable. Low-energy supersymmetry improves the accuracy of unification of the three 321 couplings^{13,7} and such theories encompass a “desert” between the weak scale ~ 250 GeV and the much-higher GUT scale $\sim 2 \times 10^{16}$ GeV, although minimal supersymmetric $SU(5)$ is by now ruled out¹⁴.

Recent developments in string theory are suggestive of a different strategy for unification of electroweak theory with QCD. Both the desert and low-energy supersymmetry are abandoned. Instead, the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group is embedded in a semi-simple gauge group such as $SU(3)^N$ as suggested by gauge theories arising from compactification of the IIB superstring on an orbifold $AdS_5 \times S^5/\Gamma$ where Γ is the abelian finite group Z_N ². In such nonsupersymmetric quiver gauge theories the unification of couplings happens not by logarithmic evolution¹¹ over an enormous desert covering, say, a dozen orders of magnitude in energy scale. Instead the unification occurs abruptly at $\mu = M$ through the diagonal embeddings of 321 in $SU(3)^N$ ⁸. The key prediction of such unification shifts

from proton decay to additional particle content, in the present model at $\simeq 4$ TeV.

Let me consider first the electroweak group which in the standard model is still un-unified as $SU(2) \times U(1)$. In the 331-model^{15,16} where this is extended to $SU(3) \times U(1)$ there appears a Landau pole at $M \simeq 4$ TeV because that is the scale at which $\sin^2\theta(\mu)$ slides to the value $\sin^2(M) = 1/4$. It is also the scale at which the custodial gauged $SU(3)$ is broken in the framework of¹⁷.

There remains the question of embedding such unification in an $SU(3)^N$ of the type described in^{2,8}. Since the required embedding of $SU(2)_L \times U(1)_Y$ into an $SU(3)$ necessitates $3\alpha_Y = \alpha_H$ the ratios of couplings at $\simeq 4$ TeV is: $\alpha_{3C} : \alpha_{3W} : \alpha_{3H} :: 5 : 2 : 2$ and it is natural to examine $N = 12$ with diagonal embeddings of Color (C), Weak (W) and Hypercharge (H) in $SU(3)^2, SU(3)^5, SU(3)^5$ respectively.

To accomplish this I specify the embedding of $\Gamma = Z_{12}$ in the global $SU(4)$ R-parity of the $N = 4$ supersymmetry of the underlying theory. Defining $\alpha = \exp(2\pi i/12)$ this specification can be made by $\mathbf{4} \equiv (\alpha^{A_1}, \alpha^{A_2}, \alpha^{A_3}, \alpha^{A_4})$ with $\Sigma A_\mu = 0 \pmod{12}$ and all $A_\mu \neq 0$ so that all four supersymmetries are broken from $N = 4$ to $N = 0$.

Having specified A_μ I calculate the content of complex scalars by investigating in $SU(4)$ the $\mathbf{6} \equiv (\alpha^{a_1}, \alpha^{a_2}, \alpha^{a_3}, \alpha^{-a_3}, \alpha^{-a_2}, \alpha^{-a_1})$ with $a_1 = A_1 + A_2, a_2 = A_2 + A_3, a_3 = A_3 + A_1$ where all quantities are defined $\pmod{12}$.

Finally I identify the nodes (as C, W or H) on the dodecahedral quiver such that the complex scalars

$$\Sigma_{i=1}^{i=3} \Sigma_{\alpha=1}^{\alpha=12} (N_\alpha, \bar{N}_{\alpha \pm a_i}) \quad (2)$$

are adequate to allow the required symmetry breaking to the $SU(3)^3$ diagonal subgroup, and the chiral fermions

$$\Sigma_{\mu=1}^{\mu=4} \Sigma_{\alpha=1}^{\alpha=12} (N_\alpha, \bar{N}_{\alpha + A_\mu}) \quad (3)$$

can accommodate the three generations of quarks and leptons.

It is not trivial to accomplish all of these requirements so let me demonstrate by an explicit example.

For the embedding I take $A_\mu = (1, 2, 3, 6)$ and for the quiver nodes take the ordering:

$$-C - W - H - C - W^4 - H^4 - \quad (4)$$

with the two ends of (4) identified.

The scalars follow from $a_i = (3, 4, 5)$ and the scalars in Eq.(2)

$$\sum_{i=1}^3 \sum_{\alpha=1}^{12} (3_\alpha, \bar{3}_{\alpha \pm a_i}) \quad (5)$$

are sufficient to break to all diagonal subgroups as

$$SU(3)_C \times SU(3)_W \times SU(3)_H \quad (6)$$

The fermions follow from A_μ in Eq.(3) as

$$\sum_{\mu=1}^4 \sum_{\alpha=1}^{12} (3_\alpha, \bar{3}_{\alpha + A_\mu}) \quad (7)$$

and the particular dodecahedral quiver in (4) gives rise to exactly *three* chiral generations which transform under (6) as

$$3[(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3})] \quad (8)$$

I note that anomaly freedom of the underlying superstring dictates that only the combination of states in Eq.(8) can survive. Thus, it is sufficient to examine one of the terms, say $(3, \bar{3}, 1)$. By drawing the quiver diagram indicated by Eq.(4) with the twelve nodes on a “clock-face” and using $A_\mu = (1, 2, 3, 6)$ I find five $(3, \bar{3}, 1)$ ’s and two $(\bar{3}, 3, 1)$ ’s implying three chiral families as stated in Eq.(8).

After further symmetry breaking at scale M to $SU(3)_C \times SU(2)_L \times U(1)_Y$ the surviving chiral fermions are the quarks and leptons of the SM. The appearance of three families depends on both the identification of modes in (4) and on the embedding of $\Gamma \subset SU(4)$. The embedding must simultaneously give adequate scalars whose VEVs can break the symmetry spontaneously to (6). All of this is achieved successfully by the choices made. The three gauge couplings evolve for $M_Z \leq \mu \leq M$. For $\mu \geq M$ the (equal) gauge couplings of $SU(3)^{12}$ do not run if, as conjectured in ^{2,8} there is a conformal fixed point at $\mu = M$.

The basis of the conjecture in ^{2,8} is the proposed duality of Maldacena¹ which shows that in the $N \rightarrow \infty$ limit $N = 4$ supersymmetric $SU(N)$ gauge theory, as well as orbifolded versions with $N = 2, 1$ and 0^{18,19} become conformally invariant. It was known long ago that the $N = 4$ theory is conformally invariant for all finite $N \geq 2$. This led to the conjecture in ² that the $N = 0$ theories might be conformally invariant, at least in some case(s), for finite N . It should be emphasized that this conjecture cannot be checked purely within a perturbative framework²⁰. I assume that the local $U(1)$ ’s which arise in this scenario and which would lead to $U(N)$ gauge groups are non-dynamical, as suggested by Witten²¹, leaving $SU(N)$ ’s.

As for experimental tests of such a TeV GUT, the situation at energies below 4 TeV is predicted to be the standard model with a Higgs boson still

to be discovered at a mass predicted by radiative corrections²² to be below 267 GeV at 99% confidence level.

There are many particles predicted at $\simeq 4$ TeV beyond those of the minimal standard model. They include as spin-0 scalars the states of Eq.(5), and as spin-1/2 fermions the states of Eq.(7). Also predicted are gauge bosons to fill out the gauge groups of (6), and in the same energy region the gauge bosons to fill out all of $SU(3)^{12}$. All these extra particles are necessitated by the conformality constraints of^{2,8} to lie close to the conformal fixed point.

One important issue is whether this proliferation of states at ~ 4 TeV is compatible with precision electroweak data in hand. This has been studied in the related model of¹⁷ in a recent article²³. Those results are not easily translated to the present model but it is possible that such an analysis including limits on flavor-changing neutral currents could rule out the entire framework.

4. Predictivity

The calculations have been done in the one-loop approximation to the renormalization group equations and threshold effects have been ignored. These corrections are not expected to be large since the couplings are weak in the entire energy range considered. There are possible further corrections such as non-perturbative effects, and the effects of large extra dimensions, if any.

In one sense the robustness of this TeV-scale unification is almost self-evident, in that it follows from the weakness of the coupling constants in the evolution from M_Z to M_U . That is, in order to define the theory at M_U , one must combine the effects of threshold corrections (due to $O(\alpha(M_U))$ mass splittings) and potential corrections from redefinitions of the coupling constants and the unification scale. We can then *impose* the coupling constant relations at M_U as renormalization conditions and this is valid to the extent that higher order corrections do not destabilize the vacuum state.

We shall approach the comparison with data in two different but almost equivalent ways. The first is "bottom-up" where we use as input that the values of $\alpha_3(\mu)/\alpha_2(\mu)$ and $\sin^2\theta(\mu)$ are expected to be 5/2 and 1/4 respectively at $\mu = M_U$.

Using the experimental ranges allowed for $\sin^2\theta(M_Z) = 0.23113 \pm 0.00015$, $\alpha_3(M_Z) = 0.1172 \pm 0.0020$ and $\alpha_{em}^{-1}(M_Z) = 127.934 \pm 0.027$ ²² we have calculated²⁴ the values of $\sin^2\theta(M_U)$ and $\alpha_3(M_U)/\alpha_2(M_U)$ for a

range of M_U between 1.5 TeV and 8 TeV. Allowing a maximum discrepancy of $\pm 1\%$ in $\sin^2 \theta(M_U)$ and $\pm 4\%$ in $\alpha_3(M_U)/\alpha_2(M_U)$ as reasonable estimates of corrections, we deduce that the unification scale M_U can lie anywhere between 2.5 TeV and 5 TeV. Thus the theory is robust in the sense that there is no singular limit involved in choosing a particular value of M_U .

Another test of predictivity of the same model is to fix the unification values at M_U of $\sin^2 \theta(M_U) = 1/4$ and $\alpha_3(M_U)/\alpha_2(M_U) = 5/2$. We then compute the resultant predictions at the scale $\mu = M_Z$.

The results are shown for $\sin^2 \theta(M_Z)$ in ²⁴ with the allowed range²² $\alpha_3(M_Z) = 0.1172 \pm 0.0020$. The precise data on $\sin^2 \theta(M_Z)$ are indicated in ²⁴ and the conclusion is that the model makes correct predictions for $\sin^2 \theta(M_Z)$. Similarly, in ²⁴, there is a plot of the prediction for $\alpha_3(M_Z)$ versus M_U with $\sin^2 \theta(M_Z)$ held with the allowed empirical range. The two quantities plotted in ²⁴ are consistent for similar ranges of M_U . Both $\sin^2 \theta(M_Z)$ and $\alpha_3(M_Z)$ are within the empirical limits if $M_U = 3.8 \pm 0.4$ TeV.

The model has many additional gauge bosons at the unification scale, including neutral Z' 's, which could mediate flavor-changing processes on which there are strong empirical upper limits.

A detailed analysis will require specific identification of the light families and quark flavors with the chiral fermions appearing in the quiver diagram for the model. We can make only the general observation that the lower bound on a Z' which couples like the standard Z boson is quoted as $M(Z') < 1.5$ TeV ²² which is safely below the M_U values considered here and which we identify with the mass of the new gauge bosons.

This is encouraging to believe that flavor-changing processes are under control in the model but this issue will require more careful analysis when a specific identification of the quark states is attempted.

Since there are many new states predicted at the unification scale ~ 4 TeV, there is a danger of being ruled out by precision low energy data. This issue is conveniently studied in terms of the parameters S and T introduced in ²⁵ and designed to measure departure from the predictions of the standard model.

Concerning T , if the new $SU(2)$ doublets are mass-degenerate and hence do not violate a custodial $SU(2)$ symmetry they contribute nothing to T . This therefore provides a constraint on the spectrum of new states.

5. Discussion

The plots we have presented clarify the accuracy of the predictions of this TeV unification scheme for the precision values accurately measured at the Z-pole. The predictivity is as accurate for $\sin^2 \theta$ as it is for supersymmetric GUT models^{7,13,26,27}. There is, in addition, an accurate prediction for α_3 which is used merely as input in SusyGUT models.

At the same time, the accurate predictions are seen to be robust under varying the unification scale around $\sim 4\text{TeV}$ from about 2.5 TeV to 5 TeV.

One interesting question is concerning the accommodation of neutrino masses in view of the popularity of the mechanisms which require a higher mass scale than occurs in the present type of model. For example, one would like to know whether any of the recent studies in²⁸ can be useful within this framework.

In conclusion, since this model ameliorates the GUT hierarchy problem and naturally accommodates three families, it provides a viable alternative to the widely-studied GUT models which unify by logarithmic evolution of couplings up to much higher GUT scales.

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